WBCS MAIN

2019

MATHEMATICS

PAPER-I

Time Allowed: 3 Hours *Full Marks: 200*

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answer may be given either in English *or in* Bengali *but all answer must be in one and the same language.*

1. Answer *any two* questions: 10×2=20

(a) Prove that $S = \{(x, y, z, w) \in \mathbb{R}^4;\ 2x + y + 3z + w = 0, x + 2y + z + 3w = 0\}$ is a vector space. Find its dimension and a basis.

(b) A is a non-singular square matrix of order 3 such that sum of the element of each row is K. Hence show that $(1, 1, 1)^T$ is an eigenvector of A. Hence show that sum of the elements in each row is $\frac{1}{k}$.

(c) If $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, show that 2 is an eigenvalue of A. Find its algebraic and geometric multiplicity. Hence comment on diagonalisability of A.

2. Answer *any two* questions: 10×2=20

(a) If the system of equations $ax + by + cz = 0; bx + cy + az = 0; cx + ay + bz = 0$ has a non-zero solution, then prove that either $a = b = c$ or $a + b + c = 0$.

(b) Find the nature of the quadratic form $Q = xy + yz + zx$ and obtain a non-singular transformation which reduce it into normal form.

(c) If S be a skew Harmitian matrix then show that $(I + S)(I - S)^{-1}$ is a unitary matrix.

3. Answer *any two* questions: 10×2=20

(a) Prove that between two real numbers there lie a rational and an irrational number.

(b) Prove that $\left\{\frac{x^n}{\lfloor n}\right\}$ is converges to $0\ \forall x \in \mathbb{R}$.

(c) For a sequence $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, prove that $\{a_n\}$ is monotone increasing and divergent. Is $\sum \frac{1}{n}$ convergent? — Justify.

 Please turn over

4. Answer *any two* questions: 10×2=20

(a) Show that $f: [0,1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 2x\sin\frac{1}{x^2} - \frac{2}{x}\cos\frac{1}{x^2}, & 0 < x \leq 1 \\ 0 & , \quad x = 0 \end{cases}$$

is *N* integrable but not R-integrable in $[0, 1]$.

(b) Show that the sequence of function $\{x^n\}$ $x \in [0, 1]$ is not uniformly convergent.

(c) Find interval of convergence of power series $\sum a_n x^n$ where $a_n = \frac{1}{(n+1)^2}$. Also comment on uniform convergence of this power series.

5. Answer *any two* questions: 10×2=20

(a) Using Lagrange Mean Value Theorem, prove that any chord of the parabola $y = ax^2 + bx + c$ is parallel to the tangent at the point whose abscissa is same as the middle point of the chord.

(b) If $f'(x)$ exist in $[0, 1]$ prove that $f(1) - f(0) = \frac{f'(x)}{2x}$ has at least one solution in $(0, 1)$.

(c) Find the envelop of the circles drawn upon the radius vectors of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as diameter.

6. Answer *any two* questions: 10×2=20

(a) Reduce the equation $x^2 - 6xy + y^2 - 10x - 10y - 19 = 0$ into canonical form and hence find the nature of the conic.

(b) Show that six normals can be drawn from a point to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(c) Prove that locus of the point of intersection of the normals to the parabola $y^2 = 4ax$ at the extremities of the focal chord is the parabola $y^2 = a(x - 3a)$.

7. Answer *any two* questions: 10×2=20

(a) Show that the lines $\frac{x-a_1}{a_2} = \frac{y-b_1}{b_2} = \frac{z-c_1}{c_2}$ and $\frac{x-a_2}{a_1} = \frac{y-b_2}{b_1} = \frac{z-c_2}{c_1}$ will intersect. Find their point of intersection.

(b) The points (0, 1, 0) and (3, –5, 2) are end points of a diameter of a sphere *S*. Find equation of the sphere on which intersection of the plane $5x - 2y + 4z + 7 = 0$ with the given sphere *S* is a great circle.

(c) Find the integrating factor of

$x(3ydx + 2xdy) + 8y^4(ydx + 3xdy) = 0$ of the form $x^\alpha y^\beta$ and hence solve it.

8. Answer *any two* questions: 10×2=20

(a) Find general and singular solution of

$$p^2(x^2 - a^2) - 2pxy + y^2 - b^2 = 0, \text{ where } p = \frac{dy}{dx}.$$

(b) Solve: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cosh x$

(c) Solve by Charpit method: $(p^2 + q^2)y = qz$.

9. Answer *any two* questions: 10×2=20

(a) Find the condition of astatic equilibrium in 2-dimension.

(b) A paraboloid of revolution is fixed with its axis vertical and vertex upward, a heavy elastic string of unstretched length $2\pi c$ is placed on it. Show that in equilibrium it rests in form of a circle of radius $\frac{4\pi ac}{4\pi a\lambda - cw}$; where w is the weight of the string, λ is modulus of elasticity and $4a$ is the latus rectum of the generating parabola.

(c) Forces X, Y, Z act along three lines $y = b, z = -c; z = c, x = -a; x = a, y = -b$ respectively show that they will have a single resultant if $\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$.

10. Answer *any two* questions: 10×2=20

(a) A particle moves in a plane under a force which is always perpendicular and towards a fixed straight line on the plane, magnitude being μ ÷ (distance from the line)^2. If initially it be at a distance $2a$ from the line and projected with a velocity $\sqrt{\frac{\mu}{a}}$ parallel to the line, prove that the path traces out by it is a cycloid.

(b) A particle describes the path $r^4 = a^4 \cos 4\theta$ under a central force. Find the law of force.

(c) State and prove Keplar's Second law on planetary motion.

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Answers may be given either in ***English*** *or in* ***Bengali*** *but all answers must be in one and the same language.*

Group A

Answer any five *questions.*

1. (a) If the equation $x^4 + px^3 + qx^2 + rx + s = 0$ has roots of the form $\alpha + i\alpha$, $\beta + i\beta$, where α, β are real. Prove that $p^2 - 2q = 0$ and $r^2 - 2qs = 0$. 14

   (b) Prove that $\sqrt{n} < \sqrt[n]{n!} < \frac{n+1}{2}$ $\forall n > 2$. 14

2. (a) If p is an odd prime prove that

   (i) $1^2.3^2.5^2 \ldots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} (mod\ p)$

   (ii) $2^2.4^2.6^2 \ldots (p-1)^2 \equiv (-1)^{\frac{p+1}{2}} (mod\ p)$ 7+7=14

   (b) (i) Show that the roots of the equation $(1+z)^n = (1-z)^n$ are the values of $i\tan\left(\frac{r\pi}{n}\right)$, where $r = 0, 1, 2, \ldots n-1$, but omitting $\frac{n}{2}$ if n is even.

   (ii) Show that the product of all values of $(\sqrt{3} + i)^{\frac{3}{5}}$ is $8i$. 7+7=14

3. (a) Define f over R^2 by

$$f(x,y) = \begin{cases} \left(\frac{|x|}{y^2}\right).e^{\frac{-|x|}{y^2}} & ,y \neq 0 \\ 0 & ,y = 0 \end{cases}$$

   Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists along any straight line but the limit does not exist. 14

   (b) Let $f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$

   Show that both f_x and f_y exist at (0, 0) but f is not differentiable at (0, 0). 7+7=14

4. (a) If V is a closed region bounded by the planes $x = 0, y = 0, z = 0, 2x + 2y + z = 4$ and $F = (3x^2 - 8z)i - 2xyj - 8xk$, then show that

   (i) $\iiint_V \nabla.F dV = \frac{16}{3}$

   (ii) $\iiint_V \nabla \times F d\nabla = -\frac{8}{3}k$ 7+7=14

(b) Proved that $a.\nabla\left(b.\nabla\left(\frac{1}{r}\right)\right) = \frac{(3(a.\vec{r})\,(b.\vec{r}))}{r^5} - \frac{a.b}{r^3}$ where a and b are constant vectors and $r = |\vec{r}|$. 14

5. (a) Let $(G, \circ)$ be a group and $(H, \circ)$ be a subgroup of $(G, \circ)$. Let $x, y \in G$ and a relation ρ is defined on G by "$x \rho y$ iff $x \circ y^{-1} \in H$." Prove that ρ is an equivalence relation on G. 14

(b) If an abelian group G of order 10 contains an element of order 5, prove that G must be cyclic group. 14

6. (a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$. Find $f(z)$ in terms of z. 14

(b) Suppose X is a non-empty set and $d(a, a) = 0$ for all $a \in X$ and $d(a, b) = 1$ for all $a, b \in X$ with $a \neq b$. Show that d is a metric on X. 14

7. (a) Construct the Lagrange interpolation polynomial for the data.

x	-1	1	4	7
$f(x)$	-2	0	63	32

Hence interpolate at $x = 5$. 14

(b) Using Newton–Raphson method solve $x \log_{10} x = 12 \cdot 34$ with $x_0 = 10$. 14

Group B

Answer any two *questions.*

8. (a) If the independent random variables X and Y be each uniformly distributed in the interval $(-a, a)$ then find the distribution of

(a) $X + Y$

(b) XY

(c) $\frac{X}{Y}$ 5+5+5=15

(b) In the equation $x^2 + 2x - q = 0, q$ is a random variable uniformly distributed over the interval (0, 2). Find the distribution function of the largest root. 15

9. (a) The least square regression lines of Y on X and X on Y are respectively $x + 3y = 0, 3x + 2y = 0$. If $\sigma_x = 1$ then find the least square regression line of V on U where $U = X + Y, V = X - Y$. 15

(b) Let $U = aX + bY$ and $V = bX - aY$. If $E(X) = E(Y) = 0$ and if $\rho(X, Y) = \rho, \rho(U, V) = 0$, then show that

(i) $\text{var}U.\text{var}V = (a^2 + b^2)^2\,(\text{var}X)(\text{var}Y)(1 - \rho^2)$

(ii) $ab(\text{var}X - \text{var}Y) = \rho\sigma_x\sigma_y(a^2 - b^2)$ 7·5+7·5=15

10. (a) By solving the dual of the primal problem:

Minimize $Z = 3x_1 - 2x_2 + 4x_3$

Subject to $3x_1 + 5x_2 + 4x_3 \geq 7,$

$6x_1 + x_2 + 3x_3 \geq 4$

$7x_1 - 2x_2 - x_3 \leq 10$

$4x_1 + 7x_2 - 2x_3 \geq 2$

$x_1 - 2x_2 + 5x_3 \geq 3, \quad x_1, x_2, x_3 \geq 0,$

Show that it has no solution. 15

(b) Five operators (A, B, C, D, E) have been assigned to five machines (I, II, III, IV, V). Operator A cannot operate machine III and operator C cannot operate machine IV. Find the optimal assignment schedule.

	I	II	III	IV	V
A	5	5	-	2	6
B	7	4	2	3	4
C	9	3	5	-	3
D	7	2	6	7	2
E	6	5	7	9	1

15